
Unstructured Computational Meshes for Subdivision Geometry of Scanned Geological Objects

Andrey A. Mezentsev¹ Antonio Munjiza² and John-Paul Latham¹

¹ Department of Earth Sciences and Engineering, Imperial College London,
London, UK (A.Mezentsev), (J.P.Latham)@imperial.ac.uk

² Department of Engineering, Queen Mary University of London, London, UK
A.Munjiza@qmul.ac.uk

Summary. This paper presents a generic approach to generation of surface and volume unstructured meshes for complex free-form objects, obtained by laser scanning. A four-stage automated procedure is proposed for discrete data sets: surface mesh extraction from Delaunay tetrahedrization of scanned points, surface mesh simplification, definition of triangular interpolating subdivision faces, Delaunay volumetric meshing of obtained geometry. The mesh simplification approach is based on the medial Hausdorff distance envelope between scanned and simplified geometric surface meshes. The simplified mesh is directly used as an unstructured control mesh for subdivision surface representation that precisely captures arbitrary shapes of faces, composing the boundary of scanned objects. CAD model in Boundary Representation retains sharp and smooth features of the geometry for further meshing. Volumetric meshes with the MezGen code are used in the combined Finite-Discrete element methods for simulation of complex phenomena within the integrated Virtual Geoscience Workbench environment (VGW).

Key words: laser scanning, unstructured mesh, mesh simplification, subdivision surfaces

1 Introduction

Recent developments in the Finite Element method (FEM) and advances in power of affordable computers have broadened the FEM application area to simulation of complex coupled phenomena in natural sciences, geology, biology and medicine in particular [Zienkiewicz]. The formulation of the combined Finite-Discrete element method (FEM-DEM) in the nineties [Munjiza] has established a connection between the continuous and discrete modeling of complex coupled phenomena. Such a formulation opens a possibility for development of integrated Virtual Prototyping Environments (VPE) in natural sciences, similar the VPE found in engineering [Latham].

VPE is typically a unification of highly inhomogeneous interacting computational components, representing the models on different levels of mathematical abstraction. For the success of VPE in natural sciences it is highly desirable to provide unified means for model representation on different levels of the models abstraction: the so-called micro and macro levels (sometimes also addressed as Mechanics of Continua and Discontinua) [Munjiza],[Latham]. For a micro level of simulation the model approximates continuous fields of system variables and systems of partial differential equations form the mathematical model. On macro level of simulation discontinuous fields of systems variables are approximated by the model and mathematically are represented by systems of ordinary or differential-algebraic equations. The FEM-DEM method is a unique computational technology, which permits representation on different levels of modeling to be combined: both micro level and macro level. Methodologically it provides a unified framework for simulations within the framework of natural sciences VPE.

As a starting point for simulation, both the FEM and the DEM require domain discretisation into a set of geometrical simplicies - a mesh. For many natural sciences applications, and specifically in geology, the main problem, making the workflow very complex, is related to absence of fully automatic methods of geometry definition and meshing. Most of the natural objects, i.e. geological particles or bio-medical entities, are characterized by complex shape that can only be captured with sophisticated scanning equipment. With increasing robustness of scanning technology it is has become possible to use realistic point-wise scanned data to define natural object geometries for simulations. Unfortunately, the output from scanned data is not usable directly for meshing and there has been much recent research reported in the area of process automation (see, for example, [Bajaj] –[Xue]).

It should be stressed that geometry definition and downstream computational mesh generation are very application specific. Moving to a new application area generally requires development of a new geometric model with different parameters, meeting specific requirements of downstream applications. Importantly, most of the developed geometric formats do not fully address discretisation requirements from the point of view of the efficiency of organization and application in the VPE, pursuing rather conflicting requirements for geometric models. The present paper addresses this problem from the point of view of CAD/mesh integration for the Virtual Geoscience Workbench environment (VGW) in natural sciences, reflecting a growing shift from stochastic to deterministic models in geological simulations.

The rest of the paper is organized as follows. In Section 2 the automatic methods for geometric models derivation based on discrete data are discussed together with basic principles of subdivision surfaces. In Section 3 a new mesh simplification concept using medial Hausdorff distance is presented. In Section 4 numerical results are given, while Section 5 gives future work and conclusions.

2 CAD Definition from Discrete Scanned Data

With the development of new scanning technology it is now possible to create large data bases of point-wise data in different areas of science and engineering. Increased accuracy of scanning permits acquisition of data sets, containing millions of data points and precisely defining the shape of different objects. Unfortunately, this information cannot be directly used in the process of the computational model defi-

dition. The size of data sets dictates development of automatic conversion methods of scanned data to geometric and further to computational models. This problem has received in recent years a lot of attention in computer vision, computational geometry and mesh generation communities [Bajaj]–[Xue].

Typically, geometry of scanned objects is defined by the boundary, using different incarnations of the so-called Boundary Representation (BREP) model. The BREP model combines surfaces with elements of topology, organized in a tree form (see, for example, [Mezentsev]–[Lang]). Most popular choices for faces underlying representation in BREP are piecewise polyhedral meshes [Owen] or spline surfaces, approximating discrete data [Lang]. As scanned data sets contain redundant data points, not corresponding to the underlying geometric complexity of the objects, initial discrete data requires simplification in most cases. Scanned discrete representation also greatly differs from application area to application area, and it changes significantly with the scanning technology used, so it is necessary to define specifics of considered input data sets.

2.1 Specifics of Scanned Data and Geometry in the Geological Applications

The development of a highly automated method of converting scanned data to surface geometry is considered. It should work with clouds of points, organized as unstructured set of X , Y , Z coordinates, type of data is common for reverse engineering, image recognition and computer visualization problems [Bajaj],[Frey]. It does not have ordered sub-parallel sliced structure, frequently found in the tomography-based bio-medical applications [Cebal]. In the considered case, the data consists of a dense bounded noisy cloud of points, lying on the boundary of the domain (see, for example Fig. 1 and Fig. 2). The discrete data have the following features [Frey]:

1. Data may be very noisy
2. Data sets are very dense (Fig. 2)
3. Straightforward approaches (like Marching Cubes [Lorensen]) frequently introduce errors of polygonal approximation, the so-called staircase effects
4. Surface reconstruction algorithms are not targeted to produce computational meshes, so the quality of meshes is low.

Addressing the specifics of the objects under consideration, it could be observed, firstly, that geological particle geometry is constrained, but not limited to a single-connected domain. It is mostly convex with random combinations of smooth rounded C_1 and highly irregular regions with sharp C_0 edges. The set up of the problem has clearly different geometry requirements in many other application areas. Secondly, data is very noisy and it is likely to have isolated scanned data points largely off-set from the reconstructed surface. This feature requires special measures to be taken to insure stability of surface reconstruction and simplification.

The problem of surface reconstruction from extremely noisy data is far from solved and a number of research papers have been published recently, addressing specific types of smooth surfaces in certain application areas (see, for example [Kolluri]). However, none of the papers address geological geometry, which requires reconstruction for the complete hull without unresolved areas, i.e. lower part of the geometry.

Thirdly, specifics of the usage of the geometrical model in the VPE simultaneously require efficiency of the geometry storage, access, rendering and meshing for

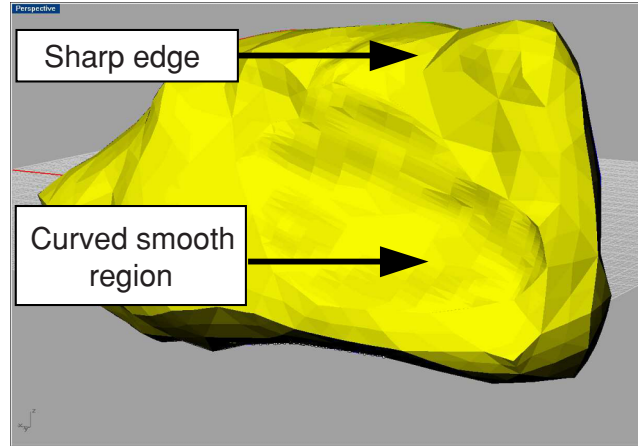


Fig. 1. Typical particle geometry as a combination of curved smooth and non-smooth sub-regions with sharp edges

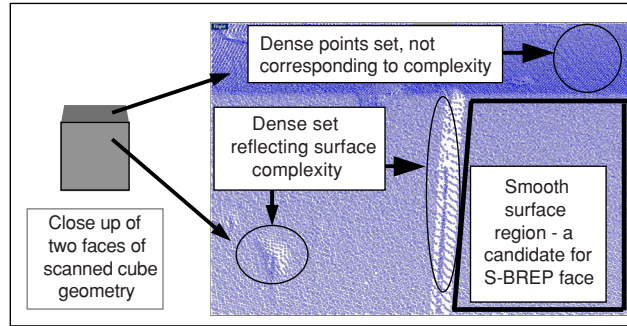


Fig. 2. Density of scanned points and geometric complexity of underlying geometry for two perpendicular faces of cube-type geometry

multiple particles at a time. Note that the VGW applications are designed to handle millions of free form particle objects with sharp and rounded features, similar to the example shown in Fig. 1.

2.2 Related Surface Reconstruction and Simplification Techniques

A comprehensive survey of recent surface reconstruction and simplification methods could be found in [Frey] and in [Kolluri]. For the sake of completeness a surface reconstruction method for the discrete data problems described above is outlined

here. Typically, surface reconstruction is a two stage process, firstly a Delaunay tetrahedrization is constructed for a set of scanned points. Secondly, a polygonal surface is extracted from volumetric discretisation using formal or heuristic techniques. In one of the most robust formal approaches, Boissonnat and Cazals [Boissonnat] successfully applied natural neighbor interpolation for surface reconstruction. Taking into consideration, that for mostly convex configurations of geological scanned particle data (see, for example, Fig. 1) outward pointing normals are known from scanning device the aforesaid method could also be used in the proposed technique. Together with interpolation of signed distance functions, proposed by Hoppe et al. [Hoppe] this method permits reconstruction of triangulated surfaces, corresponding to the initial dense and largely redundant set of scanned points. However, for the discussed geological geometry, special measures should be taken to preserve distinctive sharp features of the geometry with the C_0 continuity. The next stage of the algorithm applies a mesh simplification algorithm, based on the discrete Hausdorff distance between initial scanned points set triangulation M and simplified triangulation set M_s . Let us recall, that firstly the so-called directional Hausdorff distance can be defined as follows:

$$h(M, M_s) = \max_{m \in M} \min_{n \in M_s} \|m - n\| \quad (1)$$

Here, h - the directed Hausdorff distance from M to M_s , will be small when every point of M is close to some point of M_s . The symmetric Hausdorff distance H will be as follows:

$$H(M, M_s) = \max\{h(M, M_s), h(M_s, M)\} \quad (2)$$

However, the distances in (1) and (2) are rather fragile for a noisy scanned set. For example, a single point in M that is far enough from any point in M_s will cause h to be large. For better results, Hausdorff distances for geological scanned data requires re-formulation, reflecting possible presence of such points in the data or, alternatively, filtering of points prior to the mesh simplification may be used.

The proposed mesh simplification procedure involves iterative removal of redundant mesh nodes, not corresponding to the geometric complexity of the underlying surface. Typically, the node is removed from the mesh and resulting void is re-triangulated. Should the deviation δ (Fig. 3) of re-triangulation be within the tolerance envelope of the mesh, based on the directed Hausdorff distance H , node removal is successful. If not, the mesh node is associated with the surfaces geometric complexity and should be retained (see Fig. 3 and Fig. 4). In our case the main challenge is to develop robust technique of simplification, suitable for noisy scanned data in geological applications.

In stand-alone computational applications optimization of the simplified mesh with respect to the requirements of simulation methods produces good results [Bajaj], [Karbacher], [Frey], [Cebal]. However, the necessity of having fast geometry visualization with different levels of smoothness and details requires development of a very specific geometric model for the VGW applications, equally efficient in computer graphics and simulation related (e.g. computational mesh generation) applications. This problem is discussed in the following section.

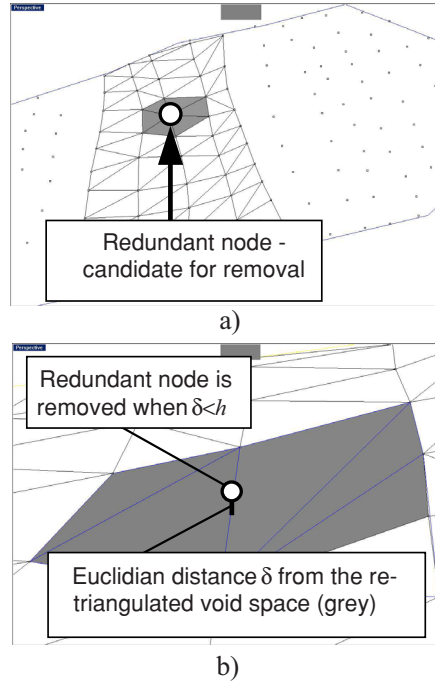


Fig. 3. Initial mesh (a), redundant node and re-triangulated simplified mesh (b)

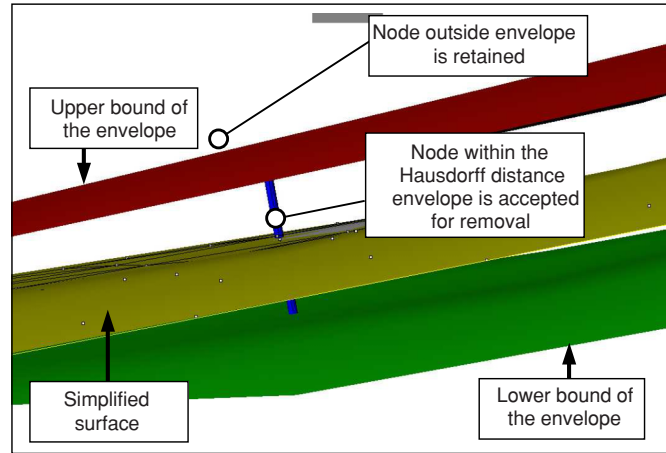


Fig. 4. Hausdorff distance envelope (upper and lower surfaces) for a given re-triangulated face, shown in Fig. 3 b), side view

2.3 Related Geometric Model

The BREP model is an efficient way of geometry definition for meshing (the topological tree of the BREP model is shown in Fig. 5). However, the BREP model should have so-called watertight properties, uniquely defining objects boundary without gaps or overlaps [Mezentsev], [Owen], [Beall]. Frequently, the classic BREP models based on splines also contain faces that are too small for quality mesh generation. The faces are unified to form bigger entities the so-called Super Elements (SE) and Constructive Elements (CE) during the BREP model meshing. From the mesh generation point of view, the best geometric model should contain only one face, covering the entire model.

Here the detailed discussion of the BREP model topology will not be given (see [Mezentsev] for details), only the concept of the subdivision BREP or S-BREP model is introduced. S-BREP is a new modification of the BREP model, with a full topological tree providing information on the hierarchy and mutual relationships of the models elements. The main building block of the S-BREP model is a face, which is based on a specific underlying surface definition. Most BREP models apply Non-Uniform Rational B-Spline (NURBS) curves and surfaces.

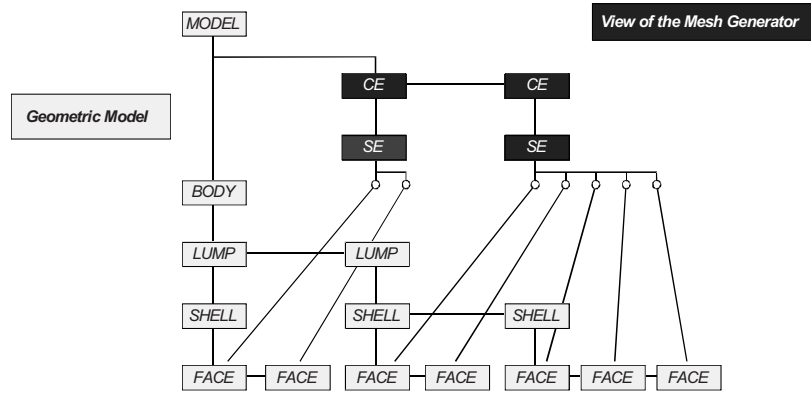


Fig. 5. Topological tree of classical BREP geometric model: CAD and mesh generation representations. Meshing typically requires coarsening of faces to unions - Super Elements (SE) and Constructive Elements (CE)

Alternatively, having both visualization and mesh generation in mind for our applications, the interpolating subdivision surface (see Zorin et al. [Zorin]) is used to define underlying face geometry in the S-BREP model. No trimming curves are required, as subdivision surface faces could be of arbitrary shape and topology. Previously interpolating subdivision surfaces have been used in mesh generation by Rypl and Bittnar [Rypl] and Lee [Lee1], [Lee2]. In [Lee1] Lee also has proposed a simplified topological model based on the subdivision surface geometry, which is further developed and generalized in present work. Let us recall main features of the subdivision surfaces.

The idea of interpolating subdivision is in representation of a smooth curve or surface as a limit of successive subdivisions of initial mesh. By starting with initial coarse mesh new positions of the inserted points are calculated according to pre-defined rules. In most cases local rules how to insert points (i.e. weighted sum of surrounding nodes coordinates [Zorin], [Dyn], [Loop]) and how to split the elements of the previous mesh are used. The resulting subdivision mesh will be smoothed out so the angles between adjacent elements will be nearly flattened. Eventually, after an infinite number of refinements, a smooth curve or surface in differential geometric sense can be obtained. For example, Fig. 6 a) shows a number of successive subdivisions for a curve. Initial coarse mesh (left, nodes are represented by hollow points) is refined by insertion of new nodes (shown as filled points) to obtain rather smooth curve representation (right).

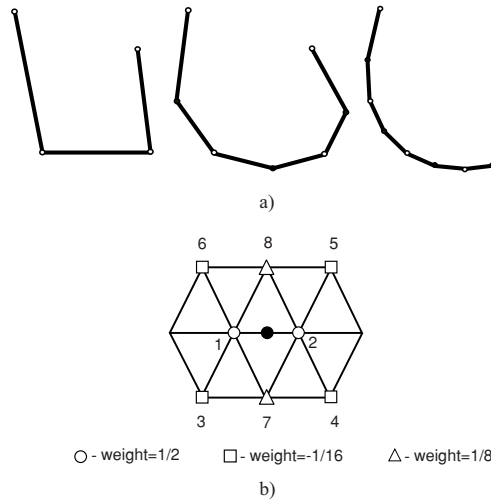


Fig. 6. Principles of subdivision: a) successive subdivisions of interpolating subdivision curve b) Butterfly subdivision scheme

The advantages of subdivision algorithms are that the schemes are local and surface representation will be good enough for most applications after a small number of refinement steps. Moreover, surface at any point can be improved arbitrarily by applying more local refinements. As a basis for the S-BREP geometry the so-called interpolating Butterfly scheme is used, proposed initially by Dyn et al. [Dyn] and later modified by Zorin et al. [Zorin]. The scheme could be applied for an arbitrary connectivity pattern of triangular mesh and uses eight points of the coarse level (Fig. 6 b), hollow points, triangles and quads) to compute position of the node on the new level of refinement (filled point, regular case). Note that the position of nodes on the previous level is retained. The following formula is used for computation of the regular node position:

$$X_p = \frac{1}{2}(X_1 + X_2) - \frac{1}{16}(X_3 + X_4 + X_5 + X_6) + \frac{1}{8}(X_7 + X_8) \quad (3)$$

For nodes with valence different from six (extraordinary internal and external nodes) different subdivision rules with different weights are applied. A complete set of rules for the modified Butterfly scheme could be found in [Zorin].

Interpolating subdivision surface is a generalization of spline surfaces for control net of arbitrary topology. Modified Butterfly scheme gives in the limit a C_1 -continuous surface and tangent vectors could be computed at any point of the surface. With reference to the triangular surface meshes considered in this study, it is also possible to apply the Loop scheme [Loop]. However, the modified Butterfly scheme produces better results on sharp corners without special topological features, producing only minor smoothing. Note that our approach models C_0 features of geometry using the tagging process, as proposed by Lee [Lee1]. For smooth regions of particle geometry the simplified geometric mesh is directly used as a control mesh of interpolating subdivision surface, sharp edges of particles are represented by discontinuities between subdivision faces.

3 Proposed Method

As it was identified in Section 2, for efficient geometry definition and storage within VGW a new definition of the Hausdorff distance for automatic scanned mesh simplification is required. It is also shown here, how the S-BREP model is constructed and utilized for mesh generation.

3.1 Medial Hausdorff Distance

For a noisy initial set of scanned points it is difficult to apply classical formulation of the Hausdorff distance (1) for initial data simplification. In pattern recognition, the modified *medial* Hausdorff distance has been introduced and successfully used (see, for example, [Chetverikov]) for pattern recognition on noisy data:

$$h^f(M, M_s) = f_{m \in M}^{th} \min_{n \in M_s} \|m - n\| \quad (4)$$

where $f_{m \in M}^{th}$ denotes the f -th quantile value of $g(x)$ over the set X for a value of f that is between zero and one. When $f = 0.5$ the so-called medial Hausdorff distance [Chetverikov] is received, which is used in our method. This measure generalizes the directed/symmetric Hausdorff distance measure, by replacing the maximum with a quantile. Medial Hausdorff distance is computed in a discrete form, similar to discrete computations in traditional formulation, proposed by Borouchaki in [Borouchaki]. Applied on a patch-wise basis, proposed modified formulation of the Hausdorff distance permits to perform filtering of the noisy data for geological and engineering applications, as demonstrated below.

3.2 Automatic Patching

Next step involves construction of the medial Hausdorff envelope on both sides of the dense scanned mesh M , as discussed in Section 2.2 and shown in Fig. 4. Together

with smoothness criterion (similar to one formulated by Frey in [Frey]) discrete normal deviation of the simplified mesh M_s is ensured to be changing smoothly. It appears, that this method works well, as its curvature modification was also successfully used in [Karbacher]. In the proposed approach normal deviation of triangles is used to automatically define patching of the whole geometric model to individual faces. For example, in Fig. 2 the smooth region on the right (outlined by the bold line) is represented by a single face, defined by underlying subdivision surface. The face is bounded by edges, naturally represented by subdivision curves. Simplified surface mesh (Section 2.2 with modifications, discussed in Section 3.1) is used directly as a control mesh for the construction of the S-BREP faces.

3.3 Computational Mesh Generation for the FEM-DEM

The problem of computational mesh generation on subdivision geometry requires further clarification. Though it is mostly reduced to the problem of surface mesh generation on the S-BREP geometric model it is worth mentioning, that *volumetric constrained* mesh generation on subdivision faces is not yet established. The methods of *surface meshing on subdivision geometry* are mainly discussed in the literature (i.e. [Rypl], [Lee1]).

In the proposed technique, two main approaches to surface meshing on the S-BREP geometry have been used for the further volumetric computational mesh generation. The first approach uses subdivision surface mesh on different levels of refinement for further constrained 3D mesh generation inside the domain, bounded by subdivision faces. The second approach to surface meshing is based on the method, proposed by Lee [Lee2] and uses parameterization of the limiting subdivision faces in the S-BREP model. It should be stressed once again, that in the S-BREP no trimming curves are required and each face is naturally bounded by the limiting boundary curves. For each face in the second approach a corresponding parametric mesh is generated, defined on the parametric $u-v$ space. In the simplest case, a parametric mesh can be obtained by face projection on one of the coordinates planes, but in more complex geometric configurations faces splitting to non-overlapping sub-domains is required. Having obtained parameterization, it is possible now to generate mesh in physical space of the S-BREP face, using traditional parametric surface mesh generation algorithms. See [Mezentsev1] for details.

MezGen unstructured Delaunay mesh generator has been applied for volumetric meshing (Mezentsev,[Mezentsev1]). Following the method, formulated in [Weatherill], the boundary recovery process in MezGen code is addressed in two main phases: an edge recovery phase and a face recovery phase. The main advantage of the MezGen approach is that rather than attempting to transform the tetrahedrons to recover edges and faces as proposed by George [George], the nodes are inserted directly into the triangulation when the edge or facet of subdivision triangulation cuts non-conforming tetrahedrons. This process temporarily adds additional nodes to the surface. Once the surface facets have been recovered, additional nodes that were inserted to facilitate the boundary recovery are deleted and the resulting local void remeshed. It is important, that in this method, no parameterization is introduced and all the operations are performed in physical space of the model. Straightforward formulation makes the method very robust, but potential pitfalls are related to the quality of direct usage of subdivision surface facets as computational meshes. That

could affect the quality of the mesh for complex faces with geometrical constraints, similar to the example in Section 4.3.

3.4 Algorithm Overview

In summary, the proposed algorithm for derivation of CAD and corresponding FEM/DEM computational models for the cloud-type discrete scanned data will be as follows:

1. Acquire discrete (point-wise) data defining object boundary from scanning device. Using current scanning technology, normal orientation is known.
2. Perform Delaunay tetrahedrization of the dense point-wise data set, using any of the appropriate algorithms, for example [Frey].
3. Reconstruct triangular surface mesh, defining scanned object, using natural neighbor interpolation method of the signed distance to the tangent hyperplanes, following [Boissonnat] and [Karbacher].
4. Simplify dense surface mesh to sparse geometric BREP mesh, using edge swapping and edge collapsing operations within the tolerance envelope of the surface, based on the modified median Hausdorff distance measure.
5. Extract smooth sub-regions (geometric faces) of the simplified surfaces mesh, using normal deviation criterion.
6. Generate faces of the S-BREP CAD model using obtained simplified mesh as a control mesh for interpolating subdivision surfaces.
7. Store S-BREP CAD model in the database of the VGW for further use.
8. Retrieve S-BREP CAD model from the data base and perform limiting subdivision to the required level of surface approximation.
9. Mesh refined subdivision CAD using constrained Delaunay methods.

It is important that based on this formulation a robust and flexible model both for visualization and computational modeling with the FEM/DEM methods is defined.

4 Numerical Results

The present implementation for the automated extraction of the described CAD model from noisy scanned cloud-of-point type data is based on the modification of the MezGen unstructured Delaunay mesh generation code [Mezentsev1] to the requirements of the VGW. VGW is a computer environment that enables the user to automate model creation for complex simulation scenarios for geosciences applications. The core component of the VGW is a virtual information space (specialized data base), unifying inhomogeneous components of different computational modules. The VGW is designed using the Object Oriented programming paradigm. This work establishes pre-processing and geometry visualization components of the VGW: scanning, data simplification and automatic S-BREP CAD creation for mesh generation.

4.1 Ellipsoid Geometry

Simple smooth point-wise ellipsoid geometry is presented in Fig. 7, a). The data was scanned with the Optix 300SE scanner (3DD Digital Corporation). Specifics of

scanning process dictate separate scanning of the upper and lower parts of the model that results in reduced model accuracy at the equator regions. This drawback is related to the obvious limitations of the technology, relying on the rotary table for scanned object movement.

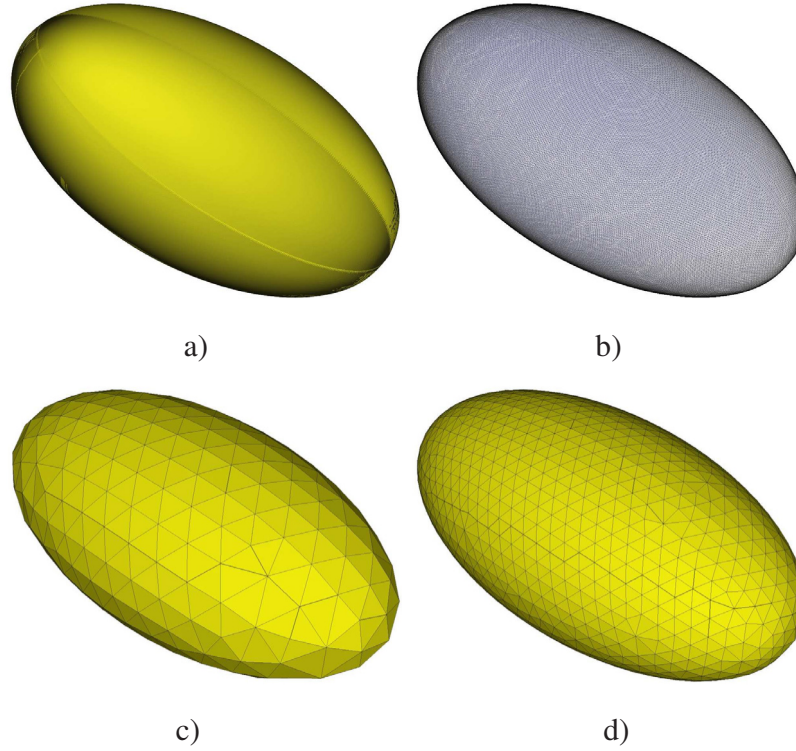


Fig. 7. Ellipsoid model: a) scanned cloud of points with visible noise b) triangulated dense cloud of points c) simplified surface mesh, used as a control mesh for subdivision surface d) computational isotropic 3D mesh on S-BREP CAD model

This test data was used to check the paradigm of automatic dense mesh (shown on Fig. 7, b)) simplification for the S-BREP limiting subdivision surface control mesh (shown on Fig. 7, c)). It appears, that for imperfect (shown on Fig. 7, a)) data set, rather noisy at the equator regions, of the scanned ellipsoid geometry, proposed method robustly captures a one-face subdivision surface in the S-BREP CAD format. The control mesh (Fig. 7, c)) has rather regular structure, simplifying application of the modified Butterfly subdivision scheme. Fig. 7 d) depicts isotropic volumetric computational mesh, generated by MezGen on the parameterized S-BREP face.

4.2 Geological Particle

Fig. 8 gives an example of the *volumetric* computational mesh (generated by the MezGen mesh generation code) on the limited subdivided S-BREP geometry of scanned particle. Our method automatically extracted 10 S-BREP faces in the form of the interpolating subdivision surfaces from a simplified subdivision mesh (shown in bold lines on Fig. 8). Only one operator intervention has been required in the process of automated face definition. Unfortunately, for free-form geometric models obtained from scanned data, minimal intervention will be required, especially for irregular rock particles with geometrically smooth and discontinuous regions. The operator driven decisions are taken when automatic algorithms fail to define CAD faces topology in one case of sharp transition between faces (Fig. 8).

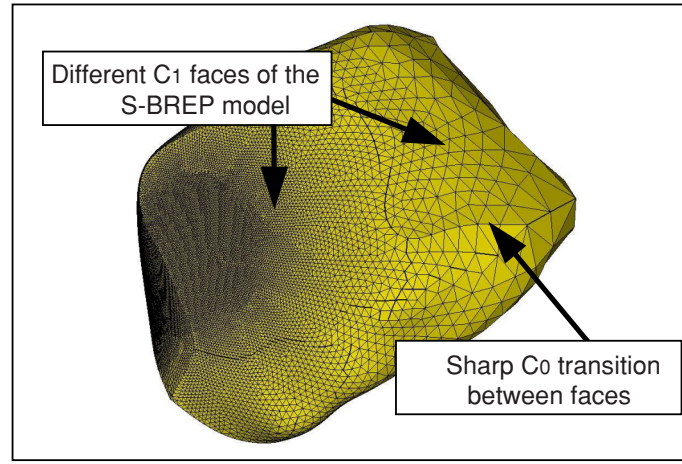


Fig. 8. Volumetric computational MezGen mesh on the limit S-BREP (subdivision surface) model of a scanned geological particle, precisely capturing the shape of object

Application-specific computational mesh refinement is also straightforward and mesh refinement shown in Fig. 8 and Fig. 9 illustrate anticipated contact area for accurate resolution of the contact forces in the FEM-DEM simulation [Munjiza].

Fig. 9 depicts volumetric computational mesh with the same level of refinement on coarse (non-limiting) subdivision geometry, practically corresponding to subdivision face control mesh after stage 4 of the described algorithm (see Section 3.4 for details).

It can be observed that both S-BREP models in Fig. 8 and Fig. 9 represent the same shape on different levels of subdivision, retaining nearly automatically extracted S-BREP faces of the geometry. This example proves applicability of the proposed method of geometry definition for meshing applications. It establishes usability of the S-BREP model with C_1 continuity of subdivision faces, combined

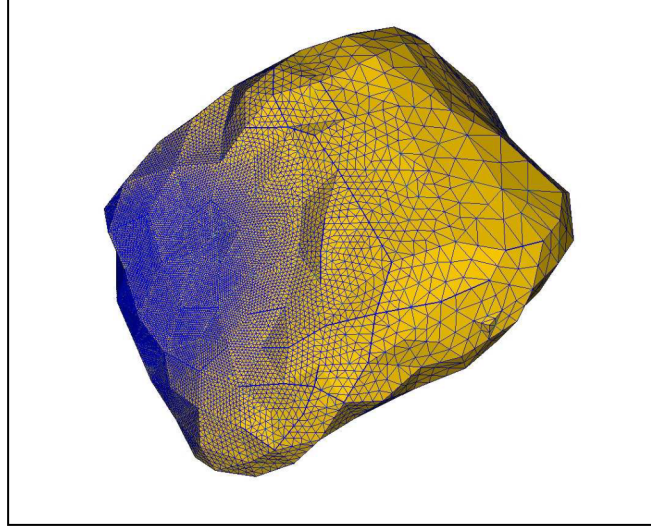


Fig. 9. Volumetric computational MezGen mesh on a coarsened S-BREP model of a scanned geological particle. Coarse facets of underlying subdivision surface are shown in bolder lines, while the quality of computational mesh is maintained (smallest dihedral angle is 11.7 degrees)

with C_0 continuity of sharp features across surface faces of free form geological particles. The other interesting observation is sharp decrease of the CAD modeling time, required for preparation of the geometric model. For the particle shown in Fig. 8 and Fig. 9 using traditional method of CAD definition with the Rhino3D package (Robert McNeel and Associates) it takes from 14 to 17 hours (depending on the operator qualification) to produce the NURBS CAD geometry. The proposed automated geometry definition and meshing method reduces this time to 2.25 hours. In terms of CAD model storage requirements new format tends to provide two times more efficient solution as compared to the Rhino3D models in ACIS (Spatial Inc.) file format.

4.3 Smooth Surface with Irregular Boundary

Earlier references related to application of interpolating subdivision surfaces for CAD modeling and computational mesh generation (e.g. [Rypl], [Lee2]) reveal certain limitations for CAD complexity, i.e. most test models were limited to a simple topology of the surface boundary. In our opinion, this is attributed to the simplified structure of applied topological model. Further development of the S-BREP overcomes this limitation, as discussed below.

In the presented example, interpolating subdivision boundary curve for the S-BREP faces is rather complex, forming highly irregular (fractal-like) boundaries. Fig. 10 a) gives scanned free form model of a surface with a regular boundary bounded by a simple four-curve outer loop. Fig. 10 b) and c) presents the same face geometry with complex irregular internal cut. Inner boundary loop is composed

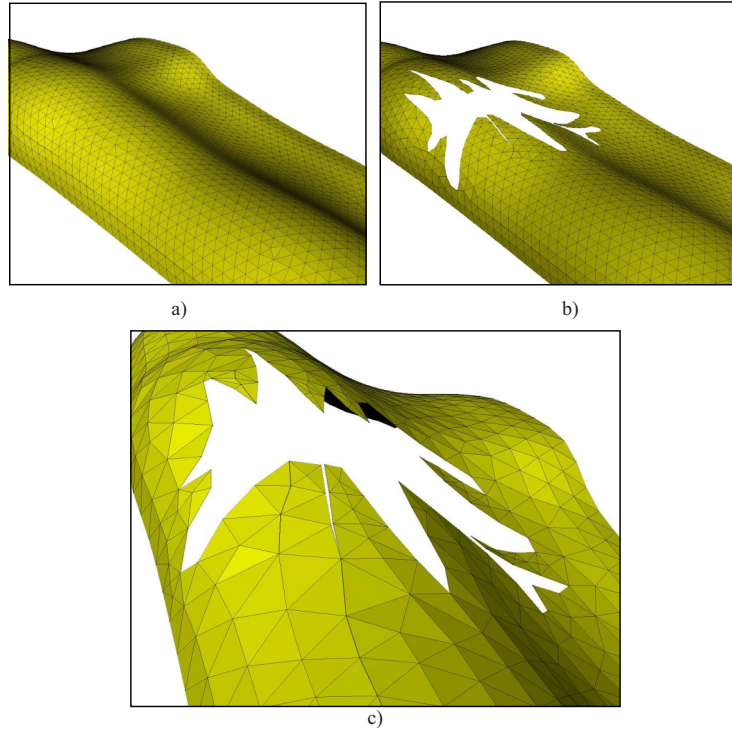


Fig. 10. Regular a) and irregular b) boundaries of free form S-BREP geometry with zoom-in on coarser level of subdivision – c)

from 14 subdivision curves, automatically connected by inserted crease extraordinary boundary node of the subdivision curve [Zorin], providing both C_0 and C_1 continuity regions on the inner boundary. This process is somewhat similar to the known effect of knot insertion into NURBS curve, creating a C_0 continuity between segments of a single spline. Given example shows, that proposed geometric model can easily integrate multiply connected domains with highly irregular boundary loops.

5 Conclusions

A new highly automated method for geometry definition and meshing of complex objects has been proposed. It is based on the methods of natural neighbors interpolation and signed distance function and a new application of the median Hausdorff distance (4) as a distance functions for automatic simplification of the scanned objects. A new concept of the S-BREP, the Boundary representation model with interpolating subdivision surfaces is introduced and implemented. The new model provides a robust alternative to the existing geometric models. It combines adaptive resolution of the geometry with automatic methods for defining faces, based on point wise data acquired from laser scanning technology.

A number of examples of different complexity show applicability and efficiency of the approach to the problem of automatic CAD definition and generation of computational meshes for noisy scanned data.

However, the problem is very complex and far from solved. More effort is required for the development of the theoretical aspects of automatic normals definition for non-convex geometry, regularization of the mapping between initial (scanned) dense and simplified surface meshes for noisy data. Further research in this area will improve the automatic extraction of faces and appropriate subdivision for VGW code development.

Acknowledgements

This work has been carried out under the Grant Virtual Geoscience Workbench, supported by the Engineering and Physical Sciences Research Council of the United Kingdom.

- [Zienkiewicz] Zienkiewicz O, Taylor R (2000) The finite element method, Volume 1. Butterworth-Heinemann, Oxford
- [Munjiza] A. Munjiza (2004) The combined discrete finite element method. Wiley, Chichester
- [Latham] Latham J-P, Munjiza A (2004) The modeling of particle systems with real shapes. *Phil Trans Royal Soc London* 362:1953–1972
- [Bajaj] Bajaj C, Bernardini F, Xu G (1996) Reconstructing surfaces and functions on surfaces from unorganized three-dimensional data. *Algorithmica* 19:362–379
- [Karbacher] Karbacher S, Laboureux X, Schon N, Hausler G (2001) Processing range data for reverse engineering and virtual reality. In: *Proceedings of the Third International Conference on 3-D Digital Imaging and Modeling (3DIM'01)*. Quebec City, Canada, 270–280
- [Boissonnat] Boissonnat J-D, Cazals F (2002) Smooth surface reconstruction via natural neighbour interpolation of distance functions. *Comp Geo* 22:185–203
- [Frey] Frey P (2004) Generation and adaptation of computational surface meshes from discrete anatomical data. *Int J Numer Meth Engng* 60:1049–1074
- [Xue] Xue D, Demkowicz L, Bajaj C (2004) Reconstruction of G1 surfaces with biquartic patches for HP Fe Simulations. In: *Proceedings of the 13th International Meshing Roundtable*, Williamsburg
- [Mezentsev] Mezentsev A, Woehler T (1999) Methods and algorithms of automated CAD repair for incremental surface meshing. In: *Proceeding of the 8th International Meshing Roundtable*, South Lake Tahoe
- [Owen] Owen S, White D (2003) Mesh-based geometry. *Int J Numer Meth Engng* 58:375–395
- [Lang] Lang P, Bourouchaki H (2003) Interpolating and meshing 3D surface grids, *Int J Numer Meth Engng* 58:209–225
- [Cebral] Cebral J, Lohner R (1999) From medical images to CFD meshes. In: *Proceeding of the 8th International Meshing Roundtable*, South Lake Tahoe
- [Lorensen] Lorensen W, Cline H (1987) Marching Cubes: a high resolution 3D surface reconstruction algorithm. *Comp Graph* 21(4):163–169
- [Kolluri] Kolluri R, Shewchuk J, O'Brien J (2004) Spectral surface reconstruction from noisy point clouds. In: *Eurographics Symposium on Geometry Processing*
- [Hoppe] Hoppe H, DeRose T, Duchamp T, McDonald J, Stuetzle W (1992) Surface reconstruction from unorganized points. *Comp Graph* 26(2):71–78

- [Beall] Beall M, Walsh J, Shepard M (2003) Accessing CAD geometry for mesh generation. In: Proceeding of the 12th International Meshing Roundtable
- [Zorin] Zorin D, Schroder P, Sweldens W (1996) Interpolating subdivision with arbitrary topology. In: Proceedings of Computer Graphic, ACM SIGGRAPH'96, 189–192
- [Ryp1] Ryp1 D, Bittnar Z (2000) Discretization of 3D surfaces reconstructed by interpolating subdivision. In: Proceeding of the 7th International Conference on Numerical Grid Generation in Computational Field Simulations, Whistler
- [Lee1] Lee C (2003) Automatic metric 3D surface mesh generation using subdivision surface geometrical model. Part 1: Construction of underlying geometrical model. *Int J Numer Meth Engng* 56:1593–1614
- [Lee2] Lee C (2003) Automatic metric 3D surface mesh generation using subdivision surface geometrical model. Part 2: Mesh generation algorithm and examples. *Int J Numer Meth Engng* 56:1615–1646
- [Dyn] Dyn N, Levin D, Gregory J (1990) A butterfly subdivision scheme for surface interpolation with tension control. In: Proceedings of Computer Graphic, ACM SIGGRAPH'90, 160–169
- [Loop] Loop C (1987) Smooth subdivision surface, based on triangles. MA Thesis, University of Utah, Utah
- [Chetverikov] Chetverikov D, Stepanov D (2002) Robust Euclidian alignment of 3D point sets. In: Proc. First Hungarian Conference on Computer Graphics and Geometry, Budapest, 70–75
- [Borouchaki] Borouchaki H (2000) Simplification de maillage basee sur la distance de Hausdorff. *Comptes Rendus de l'Académie des Sciences*, 329(1):641–646
- [Mezentsev1] Mezentsev A (2004) MezGen - unstructured hybrid Delaunay mesh generator with effective boundary recovery. <http://cadmesh.homeunix.com/andrey/Mezgen.html>
- [Weatherill] Weatherill N, Hassan O (1994) Efficient three-dimensional Delaunay triangulation with automatic point creation and imposed boundary constraints. *Int J Numer Meth Engng* 37:2005–2039
- [George] George P, Hecht F, Saltel E (1991) Automatic mesh generator with specified boundary. *Comp Meth Appl Mechan Engng* 92:269–288

